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突然扩散水跃方程的显式解

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摘要: 泄水建筑物下游的消能防冲是保证水利工程安全的重要措施。突然对称扩散水跃是水跃消能的形式之一。分析了突然扩散水跃方程(以下简称突扩水跃方程)以及突扩水跃的水力特性,研究了回流平均水深,认为回流平均水深是跃前和跃后水深的函数,同时又受突扩比的影响,提出了系数 α 的经验式,并应用动量守恒原理推导了突扩水跃方程,给出了该方程的显式解。试验验证表明,突扩水跃方程显式解与试验结果吻合,显式解与试验结果的平均误差为5.481%,说明该显式解实用可靠,精度高,可用于计算实际工程问题。

关键词: 突然扩散渠槽; 动量原理; 水跃方程; 共轭水深; 回流水深

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高水头泄水建筑物的下泄水流具有水头高、流速大、水流集中的特点,故下泄水流的动能很大。为了使下泄水流与下游河道水流正常平顺衔接,必须在泄水建筑物下游尽可能短的范围或所规定区域内集中消减下泄水流的动能,以确保建筑物与河道的安全。对于具体的水流条件、地形地质条件、泄水建筑物的形式与布置等,必须采取不同的消能方式。水跃或者底流消能作为基本的消能方式之一,已在工程中广泛应用。关于水跃消能的研究也已获得很大进展^[1-6]。

水平渠底等宽矩形断面渠道中的水跃是水跃的最简单形式,它具有共轭水深比的显式解。但是,限于地形条件,消能段经常布置成扩散式水跃消能,因此,需研究水平渠底渐扩明渠段中的水跃^[7-12]及水平突然扩散段(简称水平突扩段)的水跃^[7,13-19]。水平突扩水跃的共轭水深是底流消能工水力设计的重要参数。本文将研究突扩水跃共轭水深方程及其水力特性。首先研究回流平均水深,并通过试验资料确定回流平均水深关系中的系数 α 。然后,应用动量守恒原理推导突扩水跃方程,并给出突扩水跃方程的显式解。

1 理论分析

1.1 突扩水跃方程

图1是水平渠底突扩水跃的流动示意图。 h_1, h_2 为跃前、跃后断面水深, h_3 为回流区平均水深, b, B 为上游、下游渠槽宽度。

对图1所示的流动运用动量原理建立共轭水深关系式前,作出如下假定:①跃前与跃后断面上的水流为渐变流,动水压力服从静水压力分布;②跃前与跃后断面上的动量修正系数相等,数值为1.0;③忽略明渠

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(槽)底壁面的摩擦阻力。

在流动方向对图1的跃前与跃后两断面应用动量原理,有

$$\frac{\gamma Q}{g}(\alpha_{02}v_2 - \alpha_{01}v_1) = \sum P_i \quad (1)$$

式中: Q 为流量; γ 为水重度; g 为重力加速度; v_1 为跃前断面平均流速, v_2 为跃后断面平均流速; α_{01} , α_{02} 分别为跃前断面和跃后断面的动量修正系数; P_i 为流动方向作用的力。

根据连续性原理,有: $Q = v_1 h_1 b = v_2 h_2 B$ (2)

根据假定(1),跃前断面的动水总压力为:

$$P_1 = \frac{1}{2}\gamma h_1^2 b \quad (3)$$

跃后断面的动水总压力为: $P_2 = \frac{1}{2}\gamma h_2^2 B$ (4)

始端扩散区壁面压强与回流平均水深有关,它是确定始端扩散区壁面压强的重要参数。回流水深沿始端扩散区壁面变化。为了方便计算,通常选用平均回流水深计算水跃始端扩散断面上的动水总压力,即

$$P_3 = \frac{1}{2}\gamma h_3^2 (B - b) \quad (5)$$

根据假定(2),有: $\alpha_{01} = \alpha_{02} = 1.0$ 。 (6)

将式(2)~(6)代入式(1),得

$$\frac{\gamma Q^2}{g} \left(\frac{1}{h_2 B} - \frac{1}{h_1 b} \right) = \frac{1}{2}\gamma h_1^2 b - \frac{1}{2}\gamma h_2^2 B + \frac{1}{2}\gamma h_3^2 (B - b) \quad (7)$$

若令 $\eta = \frac{h_2}{h_1}$, $Fr_1 = \frac{Q}{\sqrt{gh_1^3 b}} = \frac{v_1}{\sqrt{gh_1}}$, $\beta = \frac{B}{b}$, 则式(7)可变成

$$\beta^2 \eta^3 + 2(1 - \beta \eta) Fr_1^2 - \beta \eta - \beta \eta (\beta - 1) \left(\frac{h_3}{h_1} \right)^2 = 0 \quad (8)$$

式(8)即为突扩水跃的共轭水深方程。

1.2 回流平均水深

由于回流水深沿始端扩散区壁面是变化的,本文认为回流平均水深 h_3 是跃前、跃后水深 h_1, h_2 的函数,同时又受突扩比 β 影响。假定

$$h_3^2 = \alpha (h_1^2 + h_2^2) \quad (9)$$

式中:系数 α 可由试验确定。根据文献[7]和[9]关于回流水深的试验资料,发现系数 α 是突扩比 β 的函数,如图2所示,其关系式可表示为

$$\alpha = -0.0669\beta^2 + 0.4846\beta - 0.329 \quad (10)$$

显然,系数 α 小于0.5,在 $\beta = 3.0$ 左右存在极值。回流水深系数由5个试验数据确定,每个试验数据取相同扩散比情况下10多个试验数据的平均值。

1.3 水跃方程显式解

将式(9)代入式(8),得

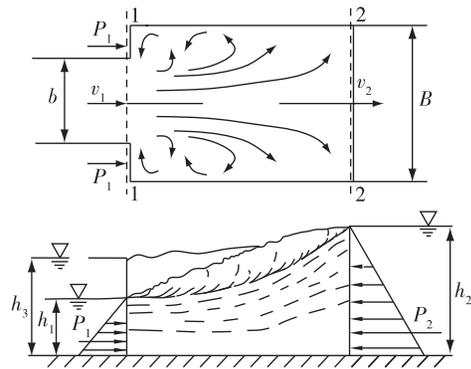


图1 突扩式水跃示意

Fig. 1 Hydraulic jump in abruptly expanding channel

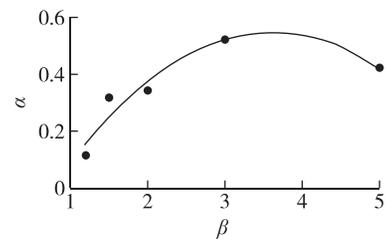


图2 系数 α 随着突扩比 β 的变化

Fig. 2 Variation of coefficient α with abrupt expansion ratio β

$$\beta^2 \eta^3 + 2(1 - \beta\eta) Fr_1^2 - \beta\eta - \alpha\beta\eta(\beta - 1)(1 + \eta^2) = 0 \tag{11}$$

整理式(11),得

$$\eta^3 - \left(\frac{2Fr_1^2 + 1 + \alpha(\beta - 1)}{\beta - \alpha(\beta - 1)}\right)\eta + \frac{2Fr_1^2}{\beta^2 - \alpha(\beta - 1)\beta} = 0 \tag{12}$$

式(12)即为假设条件下突扩水跃的共轭水深方程。对于方程(12),参照文献[20]处理一元三次方程的方法,则共轭水深方程的解为

$$\eta = 2\sqrt{\frac{2Fr_1^2 + \alpha(\beta - 1) + 1}{3[\beta - \alpha(\beta - 1)]}} \cos\varphi \tag{13a}$$

$$\varphi = \frac{1}{3} \arccos \frac{-3\sqrt{3} Fr_1^2 \sqrt{\beta - \alpha(\beta - 1)}}{\beta \sqrt{[2Fr_1^2 + 1 + \alpha(\beta - 1)]^3}} \tag{13b}$$

2 水跃方程显式解的验证

水跃方程显式解与文献[13,18]试验结果的比较见表1。

表 1 显式解与试验资料^[13,18]的比较

Tab. 1 Comparison between explicit solutions and experimental data

序号	扩散比 β	实测值			计算的跃后水深/m	误差/%	序号	扩散比 β	实测值			计算的跃后水深/m	误差/%
		跃前水深/m	跃后水深/m	跃前流速/($m \cdot s^{-1}$)					跃前水深/m	跃后水深/m	跃前流速/($m \cdot s^{-1}$)		
1	1.2	0.015	0.086	1.800	0.086	0	46	2.0	0.043	0.155	2.240	0.158	1.935
2	1.2	0.011	0.079	1.750	0.072	-8.861	47	2.0	0.027	0.163	2.347	0.133	-18.405
3	1.2	0.014	0.105	2.250	0.106	0.952	48	2.0	0.027	0.180	2.697	0.153	-15.000
4	1.2	0.022	0.151	2.550	0.149	-1.325	49	2.0	0.027	0.196	3.124	0.178	-9.184
5	1.2	0.028	0.167	2.600	0.171	2.395	50	2.0	0.027	0.226	3.658	0.209	-7.522
6	1.2	0.035	0.182	2.530	0.184	1.099	51	2.0	0.027	0.244	4.054	0.232	-4.918
7	1.2	0.043	0.196	2.480	0.199	1.531	52	2.0	0.027	0.147	1.935	0.109	-25.850
8	1.2	0.050	0.200	2.350	0.201	0.500	53	2.0	0.027	0.239	3.993	0.228	-4.603
9	1.2	0.058	0.205	2.220	0.202	-1.463	54	2.0	0.067	0.160	1.585	0.141	-11.875
10	1.2	0.011	0.096	2.280	0.096	0	55	2.0	0.067	0.203	2.042	0.180	-11.330
11	1.2	0.028	0.114	1.646	0.104	-8.772	56	2.0	0.067	0.229	2.591	0.228	-0.437
12	1.2	0.028	0.132	1.920	0.123	-6.818	57	2.0	0.067	0.272	3.109	0.275	1.103
13	1.2	0.028	0.231	3.475	0.231	0	58	2.0	0.067	0.312	3.566	0.316	1.282
14	1.2	0.028	0.157	2.377	0.155	-1.274	59	2.0	0.067	0.345	3.901	0.346	0.290
15	1.2	0.028	0.180	2.758	0.181	0.556	60	3.0	0.012	0.072	2.200	0.077	6.944
16	1.2	0.028	0.201	3.063	0.202	0.498	61	3.0	0.058	0.160	2.310	0.180	12.500
17	1.2	0.028	0.254	3.993	0.267	5.118	62	3.0	0.042	0.144	2.430	0.160	11.111
18	1.5	0.030	0.172	2.650	0.168	-2.326	63	3.0	0.030	0.130	2.480	0.137	5.385
19	1.5	0.025	0.158	2.670	0.155	-1.899	64	3.0	0.022	0.112	2.510	0.119	6.250
20	1.5	0.025	0.153	2.660	0.155	1.307	65	3.0	0.015	0.085	2.330	0.091	7.059
21	1.5	0.027	0.140	2.360	0.142	1.429	66	3.0	0.015	0.080	1.990	0.078	-2.500
22	1.5	0.026	0.107	1.890	0.110	2.804	67	3.0	0.011	0.075	2.280	0.076	1.333
23	1.5	0.016	0.105	2.100	0.098	-6.667	68	3.0	0.028	0.090	1.870	0.100	11.111
24	1.5	0.045	0.137	1.830	0.138	0.730	69	3.0	0.024	0.095	2.100	0.104	9.474
25	1.5	0.041	0.185	2.550	0.187	1.081	70	3.0	0.032	0.119	1.533	0.089	-25.210
26	1.5	0.032	0.164	2.510	0.164	0	71	3.0	0.032	0.142	2.155	0.123	-13.380
27	1.5	0.015	0.115	2.490	0.113	-1.739	72	3.0	0.032	0.165	2.481	0.141	-14.545

(续表)

序号	扩散比 β	实测值			计算的跃后水深/m	误差/%	序号	扩散比 β	实测值			计算的跃后水深/m	误差/%
		跃前水深/m	跃后水深/m	跃前流速/($m \cdot s^{-1}$)					跃前水深/m	跃后水深/m	跃前流速/($m \cdot s^{-1}$)		
28	1.5	0.027	0.132	1.814	0.107	-18.939	73	3.0	0.032	0.178	3.048	0.173	-2.809
29	1.5	0.027	0.145	2.185	0.131	-9.655	74	3.0	0.032	0.201	3.536	0.201	0
30	1.5	0.027	0.168	2.621	0.158	-5.952	75	3.0	0.032	0.221	3.871	0.220	-0.452
31	1.5	0.027	0.193	3.139	0.190	-1.554	76	3.0	0.032	0.234	4.206	0.239	2.137
32	1.5	0.027	0.216	3.597	0.219	1.389	77	3.0	0.043	0.160	1.695	0.115	-28.125
33	1.5	0.027	0.239	3.993	0.244	2.092	78	3.0	0.043	0.175	2.149	0.143	-18.286
34	1.5	0.027	0.259	4.359	0.267	3.089	79	3.0	0.043	0.203	2.667	0.177	-12.808
35	1.5	0.027	0.104	1.457	0.085	-18.269	80	3.0	0.043	0.226	3.292	0.217	-3.982
36	1.5	0.027	0.156	2.457	0.147	-5.769	81	3.0	0.043	0.249	3.658	0.242	-2.811
37	2.0	0.012	0.082	2.200	0.083	1.220	82	3.0	0.043	0.287	4.328	0.286	-0.348
38	2.0	0.034	0.153	2.520	0.159	3.922	83	5.0	0.029	0.102	2.400	0.102	0
39	2.0	0.044	0.156	2.270	0.162	3.846	84	5.0	0.060	0.132	2.300	0.144	9.091
40	2.0	0.030	0.140	2.490	0.147	5.000	85	5.0	0.039	0.123	2.450	0.121	-1.626
41	2.0	0.025	0.135	2.530	0.137	1.481	86	5.0	0.030	0.083	1.850	0.081	-2.410
42	2.0	0.013	0.094	2.560	0.101	7.447	87	5.0	0.025	0.092	2.320	0.091	-1.087
43	2.0	0.028	0.141	2.510	0.144	2.128	88	5.0	0.044	0.123	2.410	0.127	3.252
44	2.0	0.016	0.100	2.250	0.098	-2.000	89	5.0	0.058	0.135	2.270	0.140	3.704
45	2.0	0.012	0.069	1.900	0.071	2.899	90	5.0	0.023	0.100	2.600	0.098	-2.000

不同突扩比情况下水跃方程显式解与文献[13,18]试验结果的平均误差和最大误差分析见表2。计算共轭水深比与试验共轭水深比的比较见图3。可以看出:①水跃方程显式解与试验的平均误差为5.481%,最大误差为28.125%,最大误差发生在 $\beta=3.0$ 的情况。②不同突扩比情况下水跃方程显式解与文献[13,18]试验结果的比较说明,最大平均误差发生在 $\beta=3.0$ 的情况下,最小平均误差发生在 $\beta=1.2$ 的情况。最大误差和不同突扩比情况下,最大平均误差都发生在 $\beta=3.0$ 的情况,说明这种情况流动最复杂,个别试验点的试验情况和理论假定偏离较大。另外,也不排除个别试验点在试验观测时存在较大误差。③在90组试验数据中,水跃方程显式解与试验的误差小于5%的有57组,占试验资料的63.3%。误差大于20%的仅有3组,占试验资料的3.3%。这说明,水跃方程显式解是可靠的,具有较高精度,可用于计算实际工程问题。

表 2 不同突扩比时显式解与试验结果的误差
Tab. 2 Errors between explicit solution at different abrupt expansion ratios and experimental data

突扩比	$\beta=1.2$	$\beta=1.5$	$\beta=2.0$	$\beta=3.0$	$\beta=5.0$
平均误差/%	2.421	4.563	6.247	8.633	2.896
最大误差/%	8.861	18.939	25.850	28.125	9.091

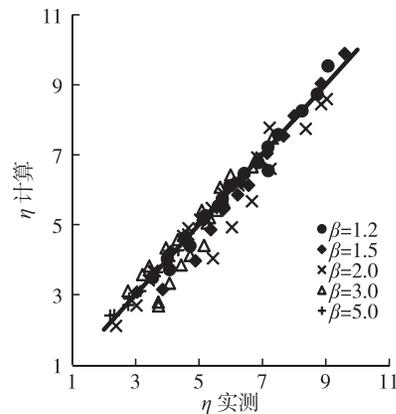


图 3 共轭水深比计算结果与试验结果比较
Fig. 3 Comparison between calculated results and experimental results of conjugate depth

3 结 语

分析研究了突扩水跃方程及其水力特性,在回流平均水深 $h_3^2 = \alpha(h_1^2 + h_2^2)$ 的假定下,用动量原理推导了突扩水跃方程,突扩水跃方程的解具有显函数形式,显式解中的系数 α 可表示为突扩比 β 函数。试验验证,突扩水跃方程显式解与试验结果的平均误差为 5.481%,最大误差为 28.125%,最大误差发生在 $\beta=3.0$ 的情况。不同突扩比的比较表明,最大平均误差发生在 $\beta=3.0$ 的情况,最小平均误差发生在 $\beta=1.2$ 的情况。在 90 组试验数据中,水跃方程显式解与试验误差小于 5% 的有 57 组,误差大于 20% 的仅有 3 组。说明该方程显式解达到了一定的精度,可用于计算实际工程问题。

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Explicit solution of hydraulic jump equation for abruptly expanding channel

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Abstract: The energy dissipation and scour prevention in the downstream of the release structures is one of the important measures to ensure the safety of hydroprojects. The abruptly expanding hydraulic jump is one of the forms of the energy dissipation of the hydraulic jump, which is widely applied to the practical hydraulic engineering. Some studies of the equations for the abruptly expanding hydraulic jump and its hydraulic characteristics including the average water depth in the backflow zone are carried out in this research. An empirical formula of coefficient α is suggested based on the assumption of the average depth $h_3^2 = \alpha(h_1^2 + h_2^2)$ in the backflow zone, and the equation of the abruptly expanding hydraulic jump is derived by the momentum conservation theory, and the explicit solution of the abruptly expanding hydraulic jump is given. The experimental results show that the average error between the explicit solution of the abruptly expanding hydraulic jump equation and the experimental data is 5.481%. Of 90 groups of the experimental data, there are 57 groups with the errors between the explicit solution of the abruptly expanding hydraulic jump equation and the experimental data being less than 5%. Obviously, the explicit solutions of the equation in this paper for the abruptly expanding hydraulic jump are in good agreement with the experimental data. It is proved that the equation in this paper for the explicit solution of the abruptly expanding hydraulic jump is reliable, practical and accurate, and can be applied to solving practical engineering problems.

Key words: abruptly expanding channel; momentum principle; hydraulic jump equation; conjugate depth; water depth in the backflow zone